4d N=1 from 6d (1,0)

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Motivation

- Compactification of 6d SCFT's can be used to better understand dynamics of 4d SCFT's.
- Has been successfully carried out for the case of the 6d SCFT living on N M5-branes, for 4d $\mathcal{N}$=2 and $\mathcal{N}$=1 SCFT's (class S).
- Recently extended also to the 6d SCFT living on N M5-branes probing a C^2/Z_k singularity, and 4d $\mathcal{N}$=1 SCFT's (class S_k).
- In this talk we will concentrate on better understanding the simpler case of $N = k = 2$. Particularly we shall present various expectations from 6d and compare against the 4d result.

Outline

1. Introduction
   - $\mathcal{N}$=2 SCFT's, $\mathcal{N}$=1 SCFT's and Class S theories
   - Class S_k theories
2. 6d perspective
3. 4d perspective
4. Conclusions

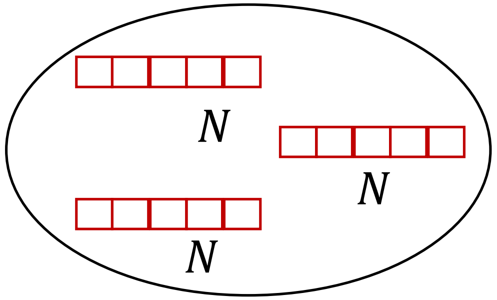
Class S theories

- Provide a systematic construction of 4d $\mathcal{N}$=2 and $\mathcal{N}$=1 SCFT's by compactification on a Riemann surface of the 6d SCFT living on N M5-branes.
- Geometric manipulations of the Riemann surface leads to interesting relations for the 4d SCFT.
- Complex structure deformation of the Riemann surface correspond to marginal operators in the 4d SCFT.
- Different realizations of the same Riemann surface imply duality relations for the 4d SCFT.

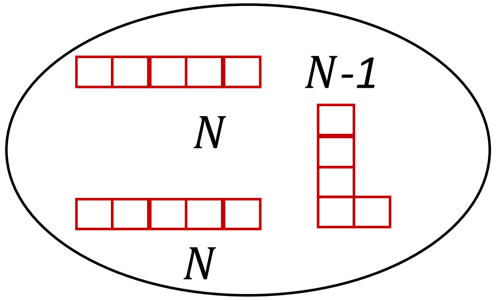
Class S theories: three punctured spheres

- Three punctured spheres play an important role in class S constructions.
- There are several types of punctures which are conveniently represented by Young diagrams.
- T_N theory: corresponds to three maximal punctures. It has an $SU(N)^3$ global symmetry.
- Bifundamental: corresponds to two maximal punctures and a minimal one.

T_N

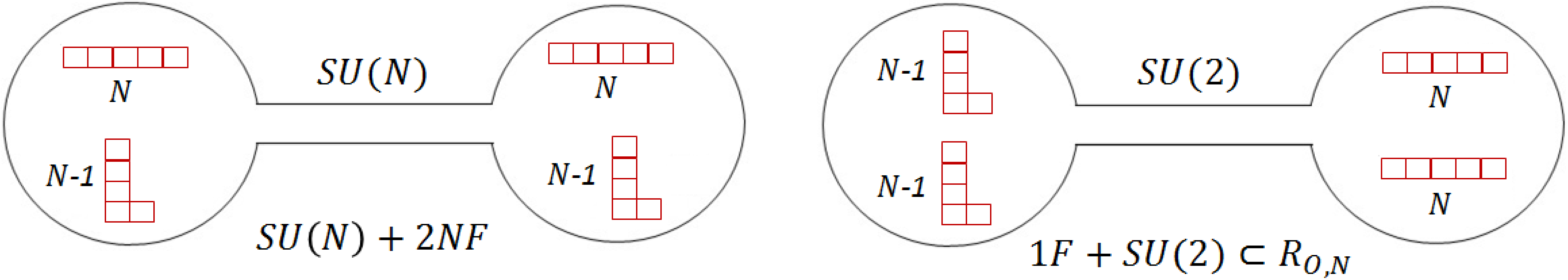


$SU(N) \times SU(N)$
Bifundamental



[Gaiotto, 2009]

Class S theories: general surfaces



- One can connect three punctured spheres with tubes to construct more general theories. Correspond to gauging the global symmetry associated to the punctures.
- The resulting 4d theory is the one associated with the surface.
- There may be more than one way to build a given surface from three punctured spheres. These different "pair of pants" decompositions correspond to dual descriptions of the same SCFT.

[Gaiotto, 2009]

6d construction

- The curvature of the Riemann surface breaks SUSY.
- In order to preserve SUSY we must perform a twist $SO(2)_S \to SO(2)_S - U(1)_R$ for $U(1)_R$ a subgroup of $U(1)_R \times SU(2)_J \subset SO(4) \subset SO(5)_R$.
- The twist makes some of the spinors covariantly constant and preserves $\mathcal{N}$=1 SUSY.
- This breaks $SO(5)_R$ to $U(1)_R \times SU(2)_J$.
- We can also give flux for the $U(1)_J$ Cartan of the $SU(2)_J$.
- This is necessary to get $\mathcal{N}$=2 SUSY, but for $\mathcal{N}$=1 we have a choice for the total flux.

6d construction

- The resulting theories are labeled by the Riemann surface and flux value.
- Gaiotto case: flux chosen to preserve $\mathcal{N}$=2. Gives the $\mathcal{N}$=2 class S theories.
- Sicillian case: flux is zero. Have an extra $SU(2)_J$ global symmetry [Benini, Tachikawa, Wecht, 2010].
- BBBW case: generic case. Have an extra $U(1)_J$ global symmetry [Bah, Beem, Bobev, Wecht, 2012].

Conformal manifold

- The conformal manifold is given by the complex structure moduli of the Riemann surface.
- In addition one can turn on flat connections, holonomies, for $SU(2)_J$ or $U(1)_J$ symmetry. These preserve only $\mathcal{N}$=1 SUSY.
- For a genus $g > 1$ Riemann surface with no punctures, we can turn on such holonomies on each of the $2g$ cycles A_i, B_j. These are subject to one relation and the action of global flavor rotation.

$$\prod_{i=1}^{g}[A_i,\, B_i] = 1$$

- For Sicilian case: $dim\mathcal{M}_g = (3g-3) + (g-1)\cdot 3 = 6g-6$
- For BBBW case: $dim\mathcal{M}_g = (3g-3) + g = 4g-3$

[Benini, Tachikawa, Wecht, 2010]
[Bah, Beem, Bobev, Wecht, 2012]

Class S_k theories

- 4d SCFT's can be constructed by the compactification of the 6d SCFT living on N M5-branes probing a C^2/Z_k singularity. This in general produces $\mathcal{N}$=1 theories.
- A class of field theories arising from this construction was conjectured in [Gaiotto, Razamat, 2015].
- Lagrangian cases can be constructed using the free trinion: corresponding to a sphere with two maximal punctures and one minimal. Is conjectured to be a collection of $2kN^2$ chiral fields.
- Maximal punctures have an $SU(N)^k$ global symmetry.
- Minimal punctures have a $U(1)$ global symmetry.
- In addition there are also $2k-1$ internal symmetries $U(1)_{\beta_i} \times U(1)_{\gamma_i} \times U(1)_t$. These are conjectured to come from the Cartan of the $SU(k) \times SU(k) \times U(1)$ global symmetry of the 6d theory.

Properties of class S_k theories

- Can construct general theories corresponding to a Riemann surface and fluxes for the internal symmetries.
- Dual frames can be generated by can be decomposing the surface as a collection of three punctured spheres connected by tubes.
- Punctures have additional discrete labels: sign, color. Associated with the charges of operators under the punctures symmetries.
  - Sign: determine the sign of the charges under $U(1)_t \times U(1)_{\beta_i} \times U(1)_{\gamma_i}$. Can be positive or negative.
  - Color: determine the charge ordering under $U(1)_{\beta_i} \times U(1)_{\gamma_i}$.

Introduction conclusion

- Rules for gluing, similar to the class S case, were conjectured in [Gaiotto, Razamat, 2015].
- These were tested by gluing free trinions.
- No direct link to the 6d construction.

Plan

- Try and connect the 4d construction to the 6d compactification.
- Consider the 6d theory on a Riemann surface. Derive expectations for the 4d theory. In this talk, concentrate on a genus > 1 Riemann surface with no puncture.
- Consider the 4d theory. Derive the same results and compare. Can do this only for the case of $N = k = 2$.

6d perspective

- Take the 6d SCFT on 2 M5-branes probing a C^2/Z_2 singularity and compactify it to 4d on a genus $g > 1$ Riemann surface.
- To preserve SUSY must twist: $SO(2)_S \rightarrow SO(2)_S - U(1)_R$, for $U(1)_R \subset SU(2)_R$.
- The $SU(k) \times SU(k) \times U(1)$ global symmetry is thought to enhance to $SO(7)$ for the $N = k = 2$ case [Ohmori, Shimizu, Tachikawa, Yonekura, 2014].
- Can also have non-zero flux in an abelian subgroup of $SO(7)$.

6d perspective: fluxes

- Decompose the flavor symmetry: $SO(7) \to SO(3) \times SO(4) \to SO(3)_t \times SU(2)_\beta \times SU(2)_\gamma$
- Define a flux vector $\mathcal{F} = (\beta, \gamma, t)$.
- The flux breaks $SO(7)$ to a subgroup G^{max} with abelian part L.
- The possible values for G^{max} are:

G^{max}	$u(1)^3$	$su(2)u(1)^2$	$su(2)_{diag}u(1)^2$	$su(2)su(2)u(1)$
L	$u(1)^3$	$u(1)^2$	$u(1)^2$	$u(1)$
$\mathcal{F}$	(a,b,c)	$(a,0,b)/(0,a,b)$	$(a,\pm a,b)$	$(a,0,0)/(0,a,0)$

G^{max}	$so(5)u(1)$	$su(3)u(1)$	$so(7)$
L	$u(1)$	$u(1)$	$\emptyset$
$\mathcal{F}$	$(0,0,a)$	$(a,0,\pm a)/(0,a,\pm a)$	$(0,0,0)$

6d perspective: conformal manifold

- Complex structure moduli.
- Can also turn on flat connections for global symmetries. Conformal manifold:

$$dim\mathcal{M}_{g,0} = \quad 3g-3+(g-1)dim\ G^{max} + dim\ L$$

- G^{max} is the maximal global symmetry on the conformal manifold.

6d perspective: anomalies

- Can calculate the 4d t' Hooft anomalies by integrating the 6d anomaly polynomial eight form.

$$\int_{\mathcal{C}_g} I_8(\mathcal{F}) = I_6^{(g,\mathcal{F})}$$

- The 6d theory anomaly polynomial was evaluated in [Ohmori, Shimizu, Tachikawa, Yonekura, 2014].
- Example for $SO(7)$ compactification:

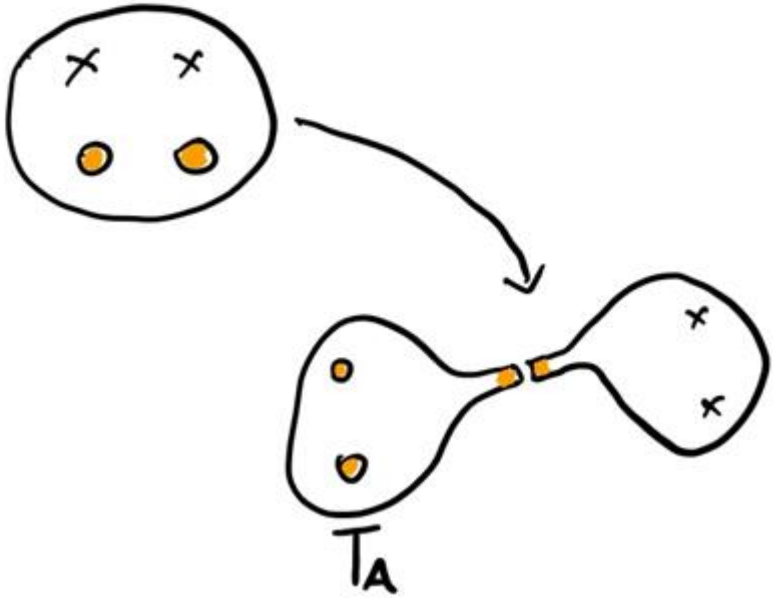
$$Tr(R^3) = 22(g-1), Tr(R) = -2(g-1)$$

$$a = \frac{3}{32}(3TrR^3 - TrR) = \frac{51}{8}(g-1)\,, \qquad c = \frac{1}{32}(9TrR^3 - 5TrR) = \frac{13}{2}(g-1)$$

4d perspective: plan

- To build the analogous theories in class S_2 we need the theories corresponding to a three punctured sphere. These are strongly interacting theories.
- We can learn about these theories using duality relations.
- Consider a duality relating a gauging of these theories to a Lagrangian theory.
- The duality can then be "inverted" leading to information on the strongly coupled theory.

4d perspective: duality



- Connecting two free trinions leads to a Lagrangian SCFT.
- This SCFT has at least two dual descriptions involving different strongly coupled three punctured spheres.

4d perspective: duality

- We find that the Lagrangian theory is dual to an $SU(2) + 2F$ gauging of the strongly interacting SCFT with 3 singlets connected via a cubic superpotential.
- The duality implies that the indices of the two theories are related as:

$$I^{Lagr} = \int d\Delta_{Haar SU(2)} I^{\text{Vector + Flavor + Singlets}} I^{\text{Interacting SCFT}}$$

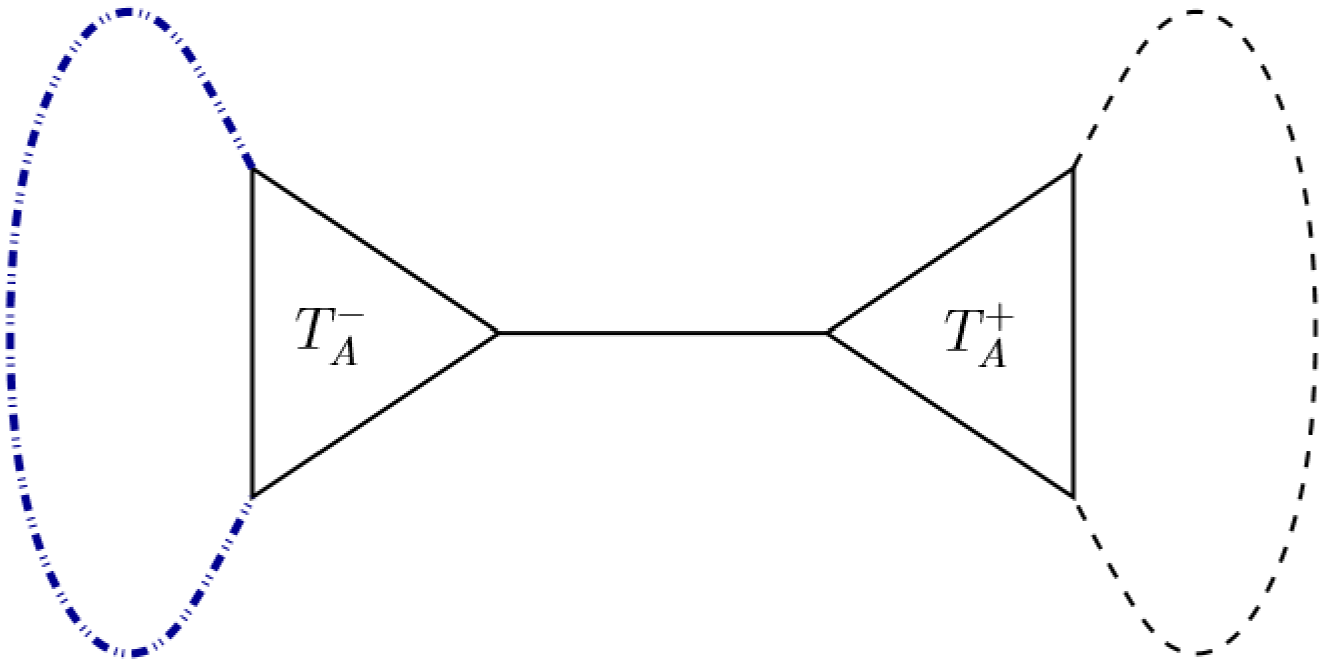
4d perspective: inversion

- There is a mathematical identity that allows to invert the relation: express the index of the strongly interacting theory as an $SU(2) + 1F$ gauging with singlets of the Lagrangian theory [Spiridonov, Warnaar, 2006].

$$I^{\text{Interacting SCFT}} = \int d\Delta_{HaarSU(2)} I'^{\text{Vector + Flavor + Singlets}} I^{Lagr}$$

- Take the further step and interpret the gauging as a physical process: duality. This gives a Lagrangian for the strongly interacting theory.
- Unfortunately Lagrangian is strongly coupled and only useful for the calculation of protected quantities: anomalies, index.

4d perspective: $SO(7)$



- Simple to generate a theory with SO(7) global symmetry by connecting a trinion to the conjugate trinion.

4d perspective: $SO(7)$

- Can evaluate the index of this theory:

$$\mathcal{I}_{g,0}^{so(7)} = 1 + \left(\left(3g - 3 + (g-1)\mathbf{21}_{so(7)}\right)\Big|_{g=2} \right) pq + \cdots .$$

- Can evaluate the conformal anomalies of this theory:

$$a = \frac{51}{8}(g-1)\,, \quad c = \frac{13}{2}(g-1)$$

4d perspective

- Can construct and match other theories. Build examples of all the G^{max} cases.
- Global symmetry, anomalies and number of marginal deformations agree with the 6d expectation.
- From matching we find: $\mathcal{F}_{T_A} = (\frac{1}{4}, \frac{1}{4}, 1), \mathcal{F}_{Free} = (0,0,\frac{1}{2})$.

Conclusions

- Matched global symmetries, dimension of conformal manifold and anomalies for selected $N = 2$ class S_2 theories against those from the 6d construction.
- Provide a non-trivial test on class S_k theories.
- Provide a bridge between field theory objects and properties of the compactification.

Open questions

- General N and k:
  - Concentrate on tori.
  - Understanding the index of class S_k theories.
  - Generalizing the inversion procedure.

Thank you